UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Ordinary Level

MARK SCHEME for the October/November 2011 question paper for the guidance of teachers

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Accuracy mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2, 1, 0 means that the candidate can earn anything from 0 to 2.

The following abbreviations may be used in a mark scheme or used on the scripts:

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy.

OW –1,2 This is deducted from A or B marks when essential working is omitted.

PA –1 This is deducted from A or B marks in the case of premature approximation.

S –1 Occasionally used for persistent slackness – usually discussed at a meeting.

EX –1 Applied to A or B marks when extra solutions are offered to a particular equation. Again, this is usually discussed at the meeting.

1	**(a)** **(i)** 7 and 0	B2	B1 for each.
	(ii) 22 and 15	B2 [4]	B1 for each.
	(b) 3 'sets' enclosed in a rectangle	B1 B1 [2]	B1 for set P and set Q separate B1 for set R contained within set P
2	$f(-2): -2a + b = 84$	M1 A1	M1 for substitution of a correct value of x
	$f\left(\frac{1}{2}\right): \frac{1}{2}a + b = \frac{3}{2}$	A1	A1 for each correct equation (allow unsimplified)
	$a = -33,\ b = 18$	M1, A1	M1 for solution to obtain a and b
	$f(1) = -19$	√B1 [6]	√B1 on their a and b
3	**(i)** Gradient $m = 4$ $\lg c = -0.6$	B1 M1 M1	M1 for a valid attempt to obtain $\lg c$ M1 for attempt to deal with $\lg c$
	$c = 0.251$	A1 [4]	
	(ii) $N = 0.251t^4$	√B1 [1]	√B1 on their m and c
4	**(i)** 6! = 720	B1 [1]	
	(ii) 2 × 5! = 240	B1 [1]	
	(iii) 4 × 5! = 480	B1 [1]	
	(iv) Even first and last: 4! (24) Odd first and even last: 4 x 4! (144) Total: 7 × 4! = 168	B1 B1 B1 [3]	

5	**(i)** $v = 2\cos 2t$ when $t = 0$, $v = 2$	M1 A1 [2]	M1 for attempt to differentiate
	(ii) $\cos 2t = 0, \quad 2t = \frac{\pi}{2}$ $t = \frac{\pi}{4}$ (0.785)	M1 A1 [2]	M1 for attempt to solve and deal with $2t$
	(iii) when $t = \frac{\pi}{4}$, $x = 4$ when $t = 0$, $x = 3$ distance moved = 1	B1 √B1 [2]	√B1 for 'their 4' –3
	(iv) $a = -4\sin 2t$ when $t = \frac{3\pi}{4}, a = 4$	M1 A1 [2]	
6	**(a)** $-5 = p + 3\tan\left(-\frac{\pi}{4}\right)$ $\therefore p = -2$ $1 = '\,p'+3\tan 3q$ $\tan 3q = 1$ $q = \frac{\pi}{12}$	M1 A1 M1 A1 [4]	M1 for use of $\left(-\frac{\pi}{12}, -5\right)$ M1for use of their p and $(q, 1)$
	(b) amplitude $a = 4$ $b = 5$ When f = 11, $x = 0$, so $c = 7$ Or when f = 3, $x = \frac{\pi}{3}$, so $c = 7$	B1 B1 M1 A1 [4]	M1 for use of either max and $x = 0$, or min and $x = \frac{\pi}{3}$

7	**(i)** $\frac{n(n-1)}{2\times 25}=\frac{3}{5}$	B1	B1 for correct term
	$n^2-n-30=0$ or $\binom{n}{2}=15$	M1	M1 equating 3rd term to $\frac{3}{5}$
		M1	M1 attempt to solve quadratic or realising that $\binom{n}{2}=15$ when $n=6$
	$n=6$	A1 [4]	
	(ii) $\left(1+nx+\frac{3}{5}x^2\right)\left(4-\frac{12}{x}+\frac{9}{x^2}\right)$		
	term : 4	B1	B1 for 4
	$-\frac{12n}{5}$ (14.4)	M1	M1 for 2nd term
	$0.18\left(n^2-n\right)$ (5.4)	M1	M1 for 3rd term
	$=-5$	A1 [4]	
8	**(a)** $\int_0^2 \mathrm{e}^{2x}+2\mathrm{e}^{x}+1\,\mathrm{d}x$	M1	M1 for expansion
	$\left[\frac{\mathrm{e}^{2x}}{2}+2\mathrm{e}^{x}+x\right]_0^2$	B1 B1 B1	B1 for each correct term
	$=41.6$	M1, A1 [6]	M1 for correct use of limits
	(b) $y=\frac{1}{2}(4x+1)^{\frac{1}{2}}(+c)$	M1	M1 for attempt to integrate
		A1	A1 for $(4x+1)^{\frac{1}{2}}$
		A1	A1 for $\frac{1}{2}(4x+1)^{\frac{1}{2}}$
	when $y=4.5$, $x=2$, $c=3$	M1	M1 for attempt to find c, must be from integration
	$y=\frac{1}{2}(4x+1)^{\frac{1}{2}}+3$	A1 [5]	A1 for $c=3$

9	**(i)**	$\text{cosec}^2 x = 8\sin x$	M1	M1 for use of correct identity or equivalent
		$\sin^3 x = \frac{1}{8}$	M1	M1 for dealing with cosec or equivalent
		$\sin x = \frac{1}{2}$	M1	M1 for attempt to solve
		$x = 30^\circ, 150^\circ$	A1, A1 [5]	Withhold last A1 if extra solutions
	(ii)	$\tan(2y - 0.3) = -\frac{5}{4}$	M1, A1	M1 for attempt to get in terms of tan
		$2y - 0.3 = 2.2455,\ 5.387$	M1	M1 for dealing with order correctly
		$y = 1.27, 2.84$ (allow 1.28 and 2.85)	A1, A1 [5]	

10 EITHER		
(i) $\frac{1}{2}(2r)^2(3\theta) - 2\frac{1}{2}r^2\theta = 5$	M1 M1	M1 for use of sector area M1 for attempt to equate area to 5
$\theta = \frac{1}{r^2}$	A1	
$P = 2r(3\theta) + 2r + 2r + 2r\theta$	M1	M1 for use of arc length
leading to $P = \frac{8}{r} + 4r$	M1	M1 for attempt to get P in terms of r and θ
(answer given)	A1 [6]	
(ii) $\frac{dP}{dr} = -\frac{8}{r^2} + 4$	M1	M1 for attempt to differentiate and equate to zero.
when $\frac{dP}{dr} = 0,\ r = \sqrt{2}$	A1	
$P = 8\sqrt{2}$	M1 A1 [4]	M1 for attempt to obtain P
(iii) $\frac{d^2P}{dr^2} = \frac{16}{r^3}$, + ve $\therefore$ minimum	B1	B1 for correct method and conclusion
when $r = \sqrt{2}, \quad \theta = \frac{1}{2}$	B1 [2]	

10	**OR**		
	(i) $OC = 10 - r$	B1 [1]	
	(ii) $\sin\theta = \frac{r}{OC}$, $\sin\theta = \frac{r}{10-r}$	M1	M1 for attempt to use sinθ
	leading to $r = \frac{10\sin\theta}{1+\sin\theta}$	A1 [2]	A1 for correct attempt to simplify to given answer
	(iii) $\frac{\mathrm{d}r}{\mathrm{d}\theta} = \frac{10\cos\theta}{(1+\sin\theta)^2}$	M1	M1 for correct attempt to differentiate a quotient
		A2, 1, 0	– 1 each error
	when $r = \frac{10}{3}$, $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$	M1	M1 for attempt to find sin or cos
		M1	M1 for substitution
	$\therefore \frac{\mathrm{d}r}{\mathrm{d}\theta} = \frac{20\sqrt{3}}{9}$ (3.85)	A1 [6]	
	(iv) $\frac{\mathrm{d}r}{\mathrm{d}t} = 2$,	B1	
	when $\theta = \frac{\pi}{6}$, $\frac{\mathrm{d}\theta}{\mathrm{d}r} = \frac{3\sqrt{3}}{20}$		
	$\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\mathrm{d}r}{\mathrm{d}t} \times \frac{\mathrm{d}\theta}{\mathrm{d}r}$	M1	M1 for correct use of rates of change
	leading to $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{3\sqrt{3}}{10}$ (0.520)	A1 [3]	